

Proof of factorization for electroproduction of multiple mesons and exclusive $\gamma^*\gamma$ production of multiple hadrons

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Abstract

In the following, we will present a generalization of the proof of factorization for electroproduction of vector mesons to the production of several mesons such as $\pi^+\pi^-$ and a proof of factorization for exclusive $\gamma^*\gamma$ production of several hadrons for example $p\bar{p}$ or $\pi^+\pi^-$ to all orders in perturbation theory up to power suppressed terms.

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I. INTRODUCTION

In the last few years several proof's of factorization for various exclusive, processes were given in [1–3], which contain novel nonperturbative functions, the so-called skewed parton distributions. This note is designed to expand this still rather meager number of processes by generalizing the proof of exclusive, electroproduction of vector mesons given in [1] to an arbitrary number of mesons [4] first discussed in [5,6], where however no all order proof was given. Furthermore, we will prove factorization to all orders in perturbation theory for exclusive $\gamma^*\gamma$ production of hadrons like $\pi^+\pi^-$ or $p\bar{p}$ [7]. All the above statements imply factorization of the amplitude of the processes under consideration up to power suppressed terms.

As will be shown, the generalization of the proof of Ref. [1] is almost trivial except for a

class of reduced diagrams which were considered in Ref. [1] but found not to contribute in ρ and J/ψ production due to C-parity. However they do contribute in, for example, $\pi^+\pi^-$ production which is now seriously being considered phenomenologically [9,10]. Furthermore, in the case of multiple meson production, the meson distribution amplitude in the factorization formula generalizes to a generalized distribution amplitude as first discussed in principle terms in [6] and in the explicit form of a two pion distribution amplitude in Ref. [9,10]. The most noteworthy and novel feature of this function is that it does not have to be real valued, i. e. , can be complex.

Exclusive $\gamma^*\gamma$ production of hadrons [8,9] is interesting in its own right, since it enables one to expand the type of processes which are sensitive to, in this case, generalized distribution amplitudes to the realm of e^+e^- colliders. In principle, these type of reactions have already been measured but the experimental cuts employed will probably have excluded such exclusive events.

The paper is organized in the following way, first we will give the generalized proof for exclusive, electroproduction of multiple mesons $\gamma_L^*(q) + P(p) \rightarrow \sum_{i=1}^N V(v_i) + P(p')$ and then the proof for exclusive $\gamma^*\gamma$ production of multiple hadrons $\gamma^*(q) + \gamma(q') \rightarrow \sum_{i=1}^N H(h_i)$ where the letters in brackets denote the momenta of the particles and H can also mean \bar{H} . Finally we will give concluding remarks.

II. PROOF OF FACTORIZATION FOR EXCLUSIVE, ELECTROPRODUCTION OF MULTIPLE MESONS

A. Statement of Factorization Theorem

The amplitude of exclusive, electroproduction of multiple mesons takes the following, factorized form

$$M = \sum_{i,j} \int_0^1 dz \int_{-1+x}^1 dx_1 P_{i/p}(x_1, x_1 - x, \mu^2) H_{ij}(x_1/x, (x_1 - x)/x, z, \mu^2) V_j(z, \zeta_1, \dots, \zeta_N, \mu^2) \\ + \text{ power suppressed terms.} \quad (1)$$

The steps in the proof of Eq. (1) are given below, following Ref. [11,12]:

- Establish the non-ultraviolet regions in the space of loop momenta contributing to the amplitude.
- Establish and proof a power counting formula for these regions.
- Determine the leading regions of the amplitude.
- Define the necessary subtractions in the amplitude to avoid double counting.
- Taylor expand the amplitude to obtain a factorized form.
- Show that the part containing the long-distance information can be expressed through matrix elements of renormalized, bi-local, gauge invariant operators of twist-2.

We will use the conventions as well as the expressions for the particle momenta of Ref. [1]. It is convenient to use light-cone coordinates with respect to the collision axis [13], the particle momenta then take the form

$$\begin{aligned}
p^\mu &= \left(p^+, \frac{m^2}{2p^+}, \mathbf{0}_\perp \right), \\
q^\mu &\simeq \left(-xp^+, \frac{Q^2}{2xp^+}, \mathbf{0}_\perp \right), \\
V^\mu &\simeq \left(\frac{\Delta_\perp^2 + m_V^2}{Q^2}, \frac{Q^2}{2xp^+}, \mathbf{\Delta}_\perp \right), \\
\Delta^\mu &\simeq \left(xp^+, -\frac{\Delta_\perp^2 + m^2 x}{2(1-x)p^+}, \mathbf{\Delta}_\perp \right).
\end{aligned} \tag{2}$$

Here, x is the Bjorken scaling variable, Q^2 is the virtuality of the initial photon, m^2 is the proton mass, $t = (p - p')^2 = \Delta^2$ is the momentum transfer squared, $V = \sum_{i=1}^N v_i$ is the sum of the momenta of the produced mesons and \simeq means “equality up to power suppressed terms”. We work in the kinematic limit of $Q^2/m_V^2 \rightarrow \infty$ with $Q^2/W^2 = \text{const.}$

Our proof is limited to a longitudinally polarized photon initiating the reaction. Photons with transverse polarization will not be considered here [14].

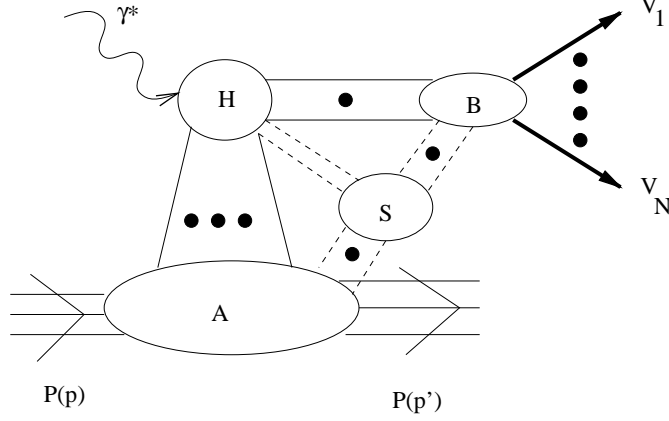


FIG. 1. Generalized reduced graph for exclusive, electroproduction of multiple mesons.

B. Regions

The steps leading to the generalized reduced graph are identical to the steps 1-3 in Sec. IV of Ref. [1]. Meaning that one first scales all momenta by a factor Q/m , secondly one uses the Coleman-Norton theorem to locate all pinch-singular surfaces in the space of loop momenta in the zero mass limit and finally identifies the relevant regions of integration as neighborhoods of these pinch-singular surfaces.

Due to the fact that one is only changing the final state produced by the collinear-to-B graph, the generalized reduced graph has to be the same as in Ref. [1], except for the final mesonic state and is given in Fig. 1. Note that, H refers to the hard subgraph of the process, A to the subgraph which is collinear to the incoming and outgoing proton, B to the subgraph which is collinear to the produced mesons and S denotes the subgraph containing lines which only have small momentum components as compared to the large scale Q . The dots represent any number of additional partons connecting the different subgraphs or additional mesons in the final state.

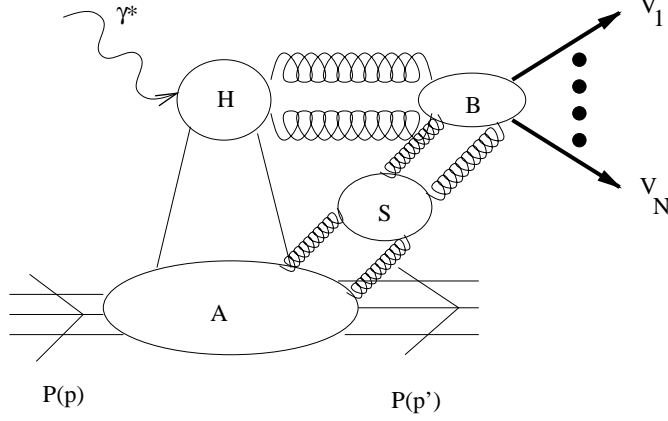


FIG. 2. Additional graph contributing to the leading region of the process.

C. Power Counting

For the same reason as mentioned in the previous subsection, the power counting formula as established in Sec. VA of Ref. [1] remains unaltered. The contribution to the amplitude from a neighborhood of a pinch-singular surface π behaves like $Q^{p(\pi)}$, modulo logarithms, in the large- Q limit with the power given by

$$p(\pi) = 3 - n(H) - \#(\text{quarks from } S \text{ to } A, B) - 3\#(\text{quarks from } S \text{ to } H) - 2\#(\text{gluons from } S \text{ to } H). \quad (3)$$

where $n(H)$ is the number of collinear quarks and transversely polarized gluons attaching to the hard subgraph H.

D. Leading regions

The leading regions are the same as the ones considered in Sec. VB of Ref. [1] and yield a leading power for the amplitude of Q^{-1} according to Eq. (3). The discussion in Sec. VB of Ref. [1] of the leading regions, as well as endpoint contributions, is unchanged since the arguments presented in [1] are independent of the number of mesons produced. Note that, as discussed below, the graph in Fig. 2 has to be reconsidered.

This leading region Fig. 2 is the same as Fig. 8c in Ref. [1], which was referred to as a glueball graph by the authors of Ref. [1]. This graph did not contribute in their analysis

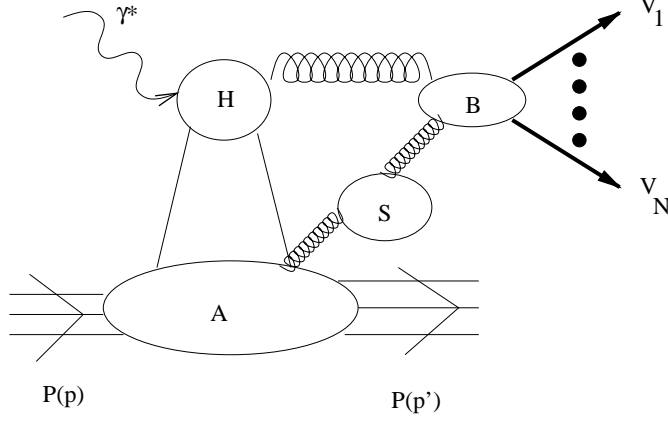


FIG. 3. Another possible leading region.

since it is zero for ρ and J/ψ production due to C-parity, but it does contribute in our case to the leading power in Q . Since this type of graph is not an endpoint-contribution-type graph, we do not have to worry about it possibly ruining the factorization of the amplitude.

Another possibility would be a graph of the type of Fig. 3. However, if the collinear-to-B gluon is a scalar gluon, i.e., has a polarization vector proportional to its momentum (k_B), one obtains the expression $k_B \cdot H$ of the scalar gluon coupling to the hard part H. Now one can use Ward identities as given in, for example, Ref. [1,15] to show that the sum over gauge invariant sets of graphs where scalar gluons connect either A to H or B to H, cancel. This is necessary since otherwise these type of graphs would be leading in Q , as compared to Fig. 2 [16].

Thus one would need transversely polarized gluons to produce one or more mesons. In this case the power in Q of the graph would be Q^0 and still leading. However, the gluon connecting H to B cannot be transversely polarized because of helicity conservation of the quarks attaching to H. The emission of a single, transverse gluon would change the helicity of the involved quark within H where the virtualities are large and helicity conservation is valid. Thus such an emission is forbidden and leads to an additional suppression by a power of Q bringing down the power in Q to Q^{-1} . Furthermore, the soft gluon would also have to have transverse polarization, which means that the operator for such a three parton state is of higher twist. This leads to an additional suppression by a factor of Q showing that the

types of graphs in Fig. 3 truly do not contribute to the leading power in Q .

E. Subtractions, Taylor expansion and gauge invariance.

The arguments given in Sec. VI of [1] about subtractions to avoid double counting in the leading regions carry through without alterations since they are very general in nature without relying on a particular final state.

The Taylor expansion of the amplitude in terms of the $+$ momentum of the collinear-to-A and $-$ momentum of the collinear-to-B partons given in SEC. VIIB and C of [1] is also unaltered, since all the arguments presented there are independent of the particulars of the final meson state.

As far as gauge invariance is concerned, the arguments presented in Sec. VIID of [1] dealing with the operators appearing in the definition of the skewed parton distributions are again unaltered since they rely on general quantum field theoretic arguments as presented in [15], independent of the final state, save its overall polarization.

Turning to the mesonic wavefunction, one immediately realizes that one is not dealing with the usual single meson wavefunction anymore but rather a multiple meson wavefunction or generalized distribution amplitude, first introduced in the case of two pion production in [9,10]. To obtain the appropriate operators for these new objects, one can follow the same line of argument as for the skewed parton distributions, and finds once more the same quark and gluon operators. Due to the additional leading region discussed in Sec. IID we also have to discuss the gluon case as compared to the situation in [1]. The definition of the generalized distribution amplitudes thus are

$$\begin{aligned} V^q(z, \zeta_i, \mu^2, m_V^2) &= \frac{1}{2N_c} \int_{-\infty}^{\infty} \frac{dy^+ e^{-izV^-y^+}}{4\pi} \langle V_1(v_1) .. V_N(v_N) | \bar{\psi}(y^+, 0, \mathbf{0}_\perp) \Gamma P \psi(0) | 0 \rangle \\ V^g(z, \zeta_i, \mu^2, m_V^2) &= -\frac{1}{2N_c} \int_{-\infty}^{\infty} \frac{dy^+ e^{-izV^-y^+}}{4\pi z V^-} \langle V_1(v_1) .. V_N(v_N) | G_\perp^-(y^+, 0, \mathbf{0}_\perp) P G^{-\perp}(0) | 0 \rangle, \end{aligned} \quad (4)$$

with z being the momentum fraction entering the generalized distribution amplitude, $\zeta_i \equiv v_i^-/V^-$, $i = 1, .., N$ characterizing the distribution of longitudinal momentum between

the mesons, μ^2 being the renormalization/ factorization scale and $\Gamma = \gamma^-, \gamma^-\gamma_5, \gamma^-\gamma^i\gamma_5$ depending on the meson species produced. Note that our definition differs from those in Ref. [9,10] in the normalization but agree with that given in [1] for a single meson wavefunction. Also note that in contrast to parton distributions or single meson wavefunctions, the above generalized distribution amplitudes are not constrained by time reversal invariance to be real valued functions anymore, since $|V_1, \dots, V_N\rangle$ differs from $\langle V_1, \dots, V_N|$ only by hermitian conjugation. As was shown in Ref. [10] in the case of the 2π distribution amplitude, the generalized distribution amplitudes develop an imaginary part above the appropriate N-particle threshold corresponding to the contribution of on-shell intermediate states. However, this does not pose a problem in the factorization proof, but is rather a phenomenological problem as shown in [10]. This is due to the fact that the above arguments are fully general and concern the whole amplitude not its real and/or imaginary parts separately. How the uncut amplitude relates to the cut one and how the different space- and time-like contributions arise or do not arise, is described in the next paragraph.

Detailing the just said, one has to keep in mind, that we are dealing with amplitudes, thus one would expect a time-ordering of the operators. This however is not necessary as shown in [18] for regular distribution amplitudes, skewed and diagonal parton distributions. The basic observation which helps prove this assertion is that the field operators are separated by a light-like distance as is also true in the above case. Thus, after a little bit of fancy footwork, it can be shown that the singularity structure in the complex plane of loop and external kinematical momenta of both the cut and uncut quark-nucleon (gluon-nucleon) scattering amplitude or the quark-anti-quark-scattering-into-a-meson amplitude is the same. This proves the equality of uncut (time-ordered) and cut (not time-ordered) amplitudes. Moreover, the singularity structure gives the appropriate support interval for the distributions. In the case of the generalized distribution amplitudes, the singularity structure is independent on the longitudinal momentum fractions of the outgoing mesons [17], thus one reobtains the case of the ordinary distribution amplitudes and thus the support interval for the generalized distribution amplitude is $0 \leq z \leq 1$ as in the case of ordinary

distribution amplitudes.

As far as evolution is concerned, the function evolves according to the usual Efremov-Radyushkin-Brodsky-LePage evolution equations. This fact can be understood with the following reasoning:

In the general case of skewed parton distributions/distribution amplitudes the evolution equations [19] depend on a skewedness parameter, basically, because the partons connecting the hard scattering subgraph H to the collinear subgraph A, carry an unequal amount of $+$ momentum, due to the exclusivity of the final state. Note that the skewedness parameter is fixed by the exclusive final state [20]. The partonic lines connecting the hard scattering subgraph H to the collinear graph B also carry unequal momentum fraction in the $-$ direction, however, in this case the skewedness is fixed to be maximal, i.e., 1 since the total $-$ momentum going into B is *fixed* by the virtual photon's $-$ momentum q^- to be $Q^2/2xp_+$, which forces one parton to have momentum fraction $(1-z)q^-$ if the other one has momentum fraction zq^- in the zero mass limit. Note that this is *not* true for the partons connecting H to A.

In the case of exclusive $\gamma^*\gamma$ production of hadrons, we will once again encounter these functions.

F. Completion of Proof

Now we can put together all of the above results and obtain Eq. (1), the generalized form of Eq. (3) of Ref. [1].

III. PROOF OF FACTORIZATION FOR EXCLUSIVE $\gamma^*\gamma$ PRODUCTION OF MULTIPLE HADRONS.

A. Statement of factorization Theorem

The amplitude of exclusive $\gamma^*\gamma$ production of multiple hadrons has the following factorized form

$$M = \sum_j \int_0^1 dz H_j(z, \mu^2) V_j(z, \zeta_1, \dots, \zeta_N, \mu^2, m_V^2) + \text{power suppressed terms.} \quad (5)$$

In the following proof, which is a generalization of the well studied factorization of $\gamma^*\gamma \rightarrow \pi^0$ [21], we will follow the same route which we outlined in Sec. II. The conventions for the momenta of the particles involved in the process are the following in light-cone coordinates [9]

$$\begin{aligned} q'^\mu &= \left(0, \frac{Q^2 + W^2}{\sqrt{2}Q}, \mathbf{0}_\perp \right), \\ q^\mu &= \left(\frac{Q}{\sqrt{2}}, -\frac{Q}{\sqrt{2}}, \mathbf{0}_\perp \right), \\ P^\mu &= \left(\frac{Q}{\sqrt{2}}, \frac{W^2}{\sqrt{2}Q}, \mathbf{0}_\perp \right), \end{aligned} \quad (6)$$

with $W^2 = P^2$ and $P = \sum_{i=1}^N h_i$. We also define, the light cone fractions as $\zeta_i \equiv h_i^+/P^+$ which differs from the ζ_i defined above in that we are now using the $+$ instead of $-$ components of the different hadron momenta. We work in the limit of $Q^2 \rightarrow \infty$ and $W^2 = \text{const.}$ which is different from the limit which we considered in the above proof where the limit is $Q^2/m_V^2 \rightarrow \infty$ with $Q^2/W^2 = \text{const.}$

B. Regions

Following the routine outlined in Sec. IIB one obtains the generalized reduced graphs which are nothing but a crossed version of the ones appearing in the proof of factorization for deeply virtual Compton scattering (DVCS) [3].

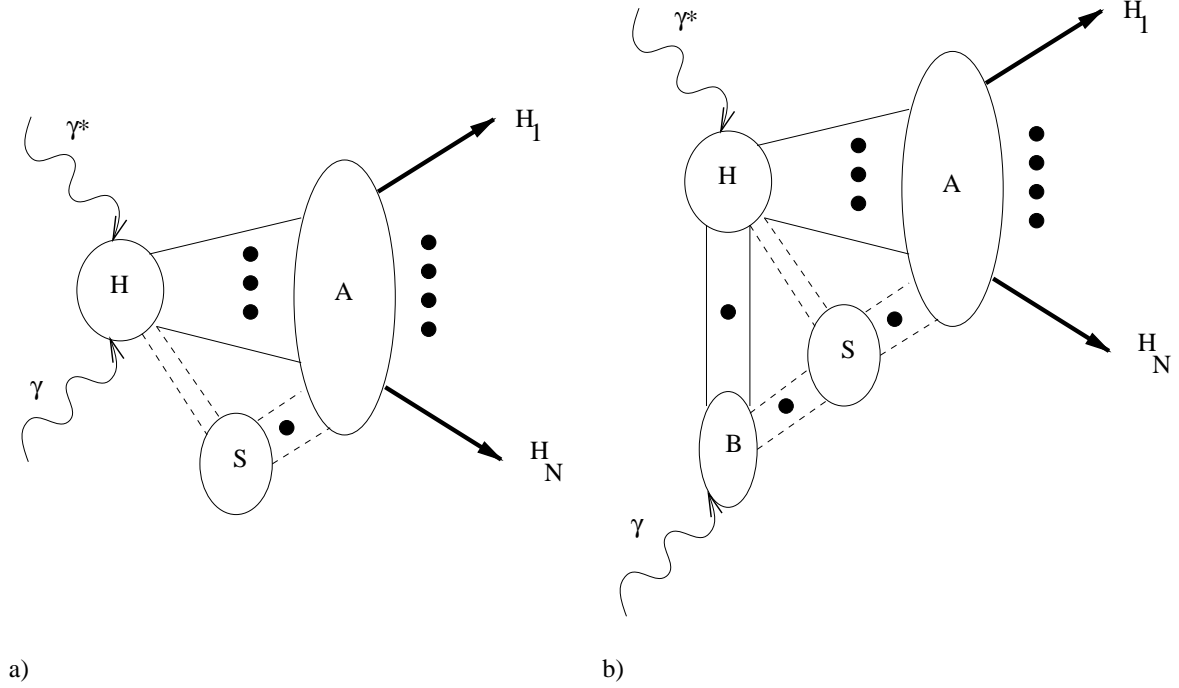


FIG. 4. Generalized reduced graphs for exclusive $\gamma^* \gamma$ production of multiple hadrons.

C. Power Counting

In the following proof of a power counting formula, we will closely follow the methodology outlined in Ref. [1]. We find the following power counting formula analogous to DVCS:

$$p(\pi) = 4 - n(H) - \#(\text{quarks from } S \text{ to } A, B) - 3\#(\text{quarks from } S \text{ to } H) - 2\#(\text{gluons from } S \text{ to } H). \quad (7)$$

where $n(H)$ is now the number of collinear quarks, transversely polarized gluons and external photons, attaching to the hard part H . The proof of Eq. (7) is exactly analogous to the one described in Ref. [1] and also employed in Ref. [3] and will thus not be repeated here for the sake of brevity. It is not too surprising that the powercounting is the same as in DVCS since the process under consideration is just a crossed version of DVCS.

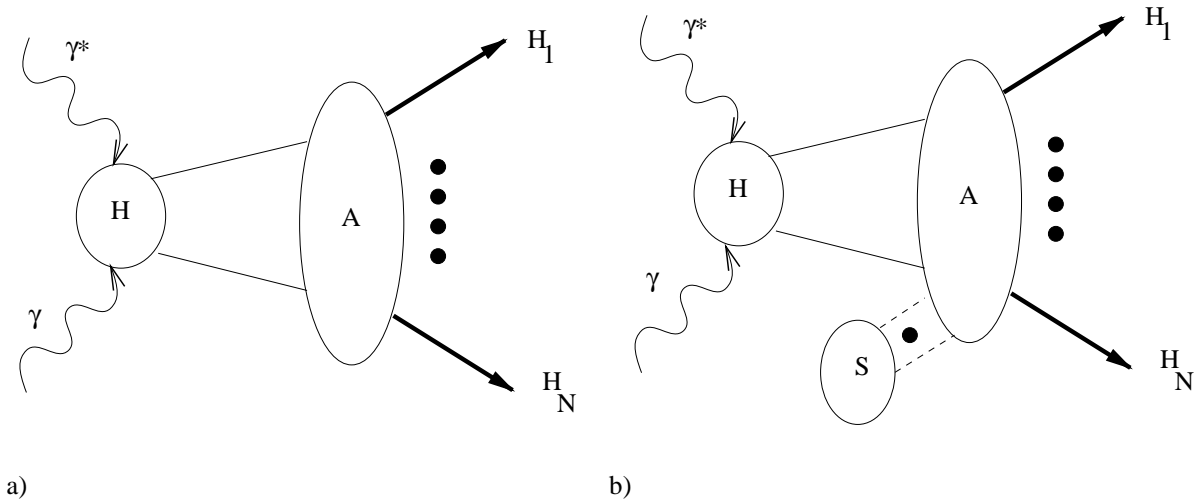


FIG. 5. Leading reduced graphs for exclusive $\gamma^*\gamma$ production of multiple hadrons.

D. Leading Regions

The leading regions are found to be those of Fig. 5, since they are the ones with the minimal number of particles attaching to H which still enables the process to proceed. It is not surprising that they are again just the crossed DVCS graphs.

Fig. 4b [22] is generally suppressed by one power of Q as compared to Fig. 4a and thus does not contribute to the leading regions, simplifying the proof of factorization tremendously as previously shown [3]. Note that this behavior is different from inclusive photoproduction where both the resolved and unresolved photon yield the same power for the cross section. The reason for this not being the case here is the following: In inclusive photoproduction there is always only one particle attaching to the hard subgraph, either the photon directly in the unresolved case or a quark/anti-quark in the resolved case, with the other quark/anti-quark going directly into the final state making a jet. If the photon is resolved in our case, however, both the quark *and* anti-quark have to attach to the hard subgraph because of the exclusiveness of the process.

Following exactly along the same lines as outlined in Ref. [3], one can easily show that there is no soft subgraph attached to the final state graph A. Hence, Fig. 5a is the only leading region in this process and the power of the amplitude for this graph, according to

our power counting formula Eq. (7), will be Q^0 , which is the same power as in DVCS.

E. Subtractions, Taylor expansion and gauge invariance

The arguments about subtractions to avoid double counting as stated in [1] carry through without change for the same reasons as mentioned in Sec. II E.

As far as the Taylor expansion of the amplitude is concerned, this carries through in the same way as stated in Ref. [3], since the structure of the amplitude in terms of subgraphs is identical to DVCS. There is a difference to the case of DVCS, however, which makes the $\gamma^*\gamma$ process much easier to deal with. Since the real photon is now in the initial state, its incoming momentum components are fixed, thus making the hard scattering function independent of *any* non-maximal skewedness parameter [23]. This in turn avoids all the difficulties encountered in DVCS when the skewedness parameter was equal to x_{Bj} [24]. However, one still has the two endpoints of the z integration, $z = 0$ and $z = 1$ to worry about since at these endpoints the lines connecting the subgraph A to H become soft and are thus not collinear-to-A anymore, possibly ruining factorization. Remember that the assumption of collinearity of the parton lines connecting A to H is essential in the proof of factorization. Note, however, that this situation is analogous to the endpoint case in DVCS [3]. There it was observed that at the endpoints, all propagators in the collinear-to-A subgraph have their poles either above or below the real axis, except one pole that runs off to either $+$ or $-\infty$, depending on which endpoint we consider, where it crosses the real axis to give a non-vanishing contribution to the amplitude. However this region where the pole crosses the real axis is a collinear-to-B region and thus power suppressed. This in turn means that the distribution amplitude vanishes at those points, thus restoring the validity of the factorization formula.

The arguments about gauge invariance as presented in Sec. VIID of [1] carry through once more without change. The operators encountered are the same as in Sec. II E when dealing with skewed parton distributions and generalized distribution amplitudes. In this

process, the out-state is obviously $\langle H(h_1)...H(h_n)|$ and the in-state is $|0\rangle$, thus we are dealing with a generalized distribution amplitude. Since, in this case, we also allow, for example, $p\bar{p}$ final states and have P in $+$ direction, we need to slightly change our notation for the generalized distribution amplitudes [25]

$$\begin{aligned} V^q(z, \zeta_i, \mu^2, m_H^2) &= \frac{1}{2N_c} \int_{-\infty}^{\infty} \frac{dy^-}{4\pi} e^{-izP^+y^-} \langle H(h_1)...H(h_n) | \bar{\psi}(0, y^-, \mathbf{0}_\perp) \Gamma P \psi(0) | 0 \rangle \\ V^g(z, \zeta_i, \mu^2, m_H^2) &= -\frac{1}{2N_c} \int_{-\infty}^{\infty} \frac{dy^-}{4\pi z P^+} e^{-izP^+y^-} \langle H(h_1)...H(h_n) | G_\perp^+(0, y^-, \mathbf{0}_\perp) P G^{+\perp}(0) | 0 \rangle, \end{aligned} \quad (8)$$

with $\Gamma = \gamma^+, \gamma^+ \gamma_5, \gamma^+ \gamma^i \gamma_5$. The comments made in Sec. II E concerning Eq. (8) are unaltered.

F. Completion of Proof

After the above said, we just have to assemble the above statements into the final factorization equation Eq. (5) to complete the proof.

IV. CONCLUSIONS

We have proved a generalization of the factorization theorem given in [1] to an arbitrary number of mesons. This generalization led us to introduce new nonperturbative objects called generalized distribution amplitudes. Furthermore, we proved a factorization theorem for $\gamma^* \gamma$ production of several hadrons to all orders in perturbation theory up to power suppressed terms.

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$$V^\mu = (V^+, V^-, V_\perp) = \left(\frac{V^0 + V^3}{\sqrt{2}}, \frac{V^0 - V^3}{\sqrt{2}}, V^{1,2} \right).$$
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- [24] See Ref. [3] for a detailed discussion on the problems associated with this particular situation.
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